647510 **MSC4MatC404x** **Seat No :________**

M.Sc.(Maths) Semester - 4 (*CBCS*) Examination
April/May-2022 (NEW COURSE)
Graph Theory (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que-1

(A) **Answer any Two:** **10**

(1) Write a short note on $\boldsymbol{K\ddot{o}nigsberg\ Bridge}$ problem.

(2) Define: Simple Graph
Prove that a simple graph with $\boldsymbol{n\ vertices\ and\ k\ components}$ can have atmost $\frac{1}{2}(\boldsymbol{n-k})(\boldsymbol{n-k+1})$ number of edges.

(3) Define: Edge disjoint subgraphs and Vertex disjoint subgraphs
Explain that a vertex disjoint subgraph is always an edge disjoint subgraph.

(B) **Answer any One:** **04**

(1) Define: Incidence Matrix.
Prove that the rank on the incidence matrix $\boldsymbol{A}(\boldsymbol{G})$ of a connected graph $\boldsymbol{G}$ with $\boldsymbol{n}$ vertices is $\boldsymbol{n-1}$.

(2) Define: Adjacency Matrix.
List the observations/properties of the adjacency matrix of a graph $\boldsymbol{G}$.

Que-2

(A) **Answer any Two:** **10**

(1) Define: Euler Graph
Prove that a connected graph $\boldsymbol{G}$ is an Euler graph if and only if it can be decomposed into circuits.

(2) Prove that in a complete graph with n vertices there are $\frac{1}{2}(n-1)$ edge-disjoint Hamiltonian circuits if $\boldsymbol{n}$ is an odd number and $\boldsymbol{n \geq 3}$.

(3) In a connected graph $\boldsymbol{G}$ with exactly $\boldsymbol{2k}$ odd vertices, there exists $\boldsymbol{k}$ edge-disjoint subgraphs such that they together contain all edges of $\boldsymbol{G}$ and that each is a unicursal graph.

(B) **Answer any One:** **04**

(1) Prove that a given connected graph $\boldsymbol{G}$ is an Euler graph if and only if all vertices of $\boldsymbol{G}$ are of even degree.

(2) Define: Circuit
Prove that a connected graph $\boldsymbol{G}$ remains connected after removing an edge $\boldsymbol{e}$ from $\boldsymbol{G}$ if and only if this edge $\boldsymbol{e}$ is in some circuit in $\boldsymbol{G}$.

Que-3

(A) **Answer any Two:** **10**

(1) Define: Edge Connectivity & Vertex Connectivity.
Prove that the vertex connectivity of any graph $\boldsymbol{G}$ can never exceed the edge connectivity of $\boldsymbol{G}$.

(2) Prove that every circuit has an even number of edges in common with any cut-set.

(3) State and prove the ***max-flow min-cut*** theorem.

(B) **Answer any One:** **04**

(1) Define: Minimally Connected Graph.
Prove that a graph $\boldsymbol{G}$ is a tree if and only if it is minimally connected.

(2) Prove that the number of vertices $\boldsymbol{n}$ in a binary tree is always odd and also prove that a binary tree has exactly $\frac{\boldsymbol{n+1}}{\boldsymbol{2}}$ pendant vertices.

Que-4

(A) **Answer any Two:** **10**

(1) **Explain**: Combinatorial Graph <u>and</u> Geometric Graph.

(2) Define: Planar Graph
Prove that Kuratowski's first graph is a non-planar graph.

(3) State and prove the Euler's formula for planar graphs.

(B) **Attempt any one:** **04**

(1) Explain the procedure for detection of planarity of a given graph G.

(2) If $\boldsymbol{G}$ be a graph with n vertices, $\boldsymbol{e}$ edges and $\boldsymbol{f}$ faces, then prove that
(i) $\boldsymbol{e \geq \frac{3}{2} f}$ (ii) $\boldsymbol{e \leq 3n - 6}$.

Que-5

(A) **Answer any Two:** **10**

(1) Define: Dominating Set.
By taking a suitable graph $\boldsymbol{G}$, find all minimal dominating sets of $\boldsymbol{G}$ by using Boolean arithmetic.

(2) Define: Maximal Independent Set.
Explain the procedure of finding **all** maximal independent sets in a connected graph by using Boolean arithmetic.

(3) Define: $\boldsymbol{\varkappa}$-chromatic graph
Prove that a tree with two or more vertices is 2-chromatic.

(B) **Attempt any one:** **04**

(1) Prove that a graph of $\boldsymbol{n}$ vertices is a complete graph ***if and only if*** its chromatic polynomial is $\boldsymbol{P_n(\lambda) = \lambda(\lambda - 1)(\lambda - 2) \cdots (\lambda - n + 1)}$.

(2) Define: (i)Chromatic Number (ii) Matching
(iii)Maximal Matching (iv) Matching Number

****